Entanglement between two multiphoton Tavis-Cummings qubits and an isolated qubit

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1. The model and its exact solution

Let us now present the model we are interested in. Consider three identical two-level natural or artificial atoms (qubits) A_1, A_2, and A_3. The atoms A_2 and A_3 are trapped in a single-mode infinite-Q cavity and resonantly interacting with the cavity field through the m-photon transitions. The atom A_1 is outside the cavity, and interaction between the field of the cavity and atom A_1 is absent. There is no direct interaction between atoms. The configuration is shown in Fig. 1.

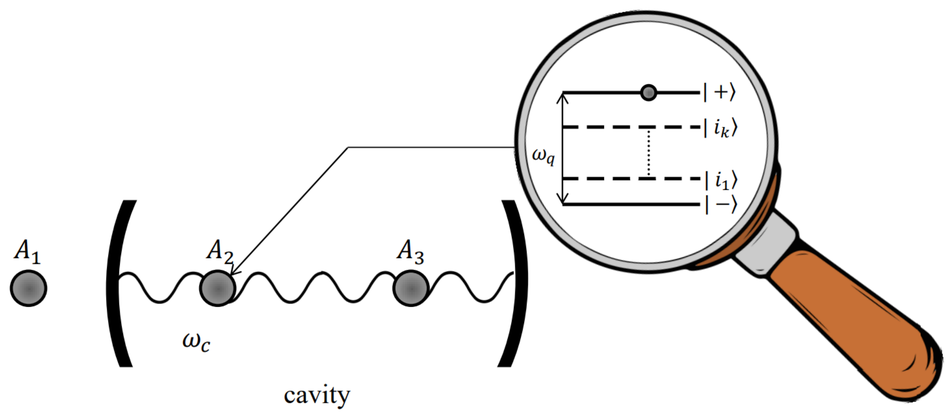

Figure 1: *Configuration of the model under study. Here, ω_c is the frequency of the resonator, $\omega_q = m\omega_c$ is the transition frequency in the qubit between the excited $|+\rangle$ and the ground $|-\rangle$ levels, m is the photon multiple of transitions, and $|i\rangle_k$ are the virtual intermediate levels, the number of which is determined by the parameter $k = m - 1$.*

The Hamiltonian of the interaction of the studied model will be written in the dipole approximation and the rotating wave approximation in the following form

$$\hat{H}_I = \hbar\gamma \sum_{i=2}^{3} \left(\hat{\sigma}_i^+ \hat{a}^m + \hat{\sigma}_i^- \hat{a}^{+m}\right), \tag{1}$$

where $\hat{a}^+(\hat{a})$ is the operator of the creation (annihilation) of photons of the resonator mode of the field, $\hat{\sigma}_i^+$ and $\hat{\sigma}_i^-$ are the raising and the lowering operators in the i-th atom, and γ is the atom-field coupling constant. Index i in (1) numbers the atoms trapped in a cavity.
As the initial state of the resonator field, we choose a thermal state with a density matrix of the form:

$$\hat{\rho}_F(0) = \sum_n p_n |n\rangle\langle n|. \tag{2}$$

There are weight coefficients

$$p_n = \frac{\langle n\rangle^n}{(1+\langle n\rangle)^{n+1}}, \quad \langle n\rangle = \frac{1}{e^{\hbar\omega_c/k_BT} - 1}$$

the average number of photons of the resonator, and T is the cavity temperature.
We derived the solutions of the quantum Liouville equation for the whole density matrix $\hat{\rho}_{A_1A_2A_3F}$ of the considered system with Hamiltonian (1)

$$i\hbar\frac{\partial\hat{\rho}_{A_1A_2A_3F}}{\partial t} = \left[\hat{H}_I, \hat{\rho}_{A_1A_2A_3F}\right], \tag{3}$$

and for initial qubit W- and GHZ-states and the thermal state of the cavity field (2).

2. Selection of initial states and calculation of concurrence, fidelity

Let's choose entangled W-type states of the form as the initial states of the atoms subsystem

$$|W_1(0)\rangle_{A_1A_2A_3} = \cos\theta|-,-,+\rangle + \sin\theta\sin\varphi|-,+,-\rangle + \sin\theta\cos\varphi|+,-,-\rangle, \tag{4}$$

$$|W_2(0)\rangle_{A_1A_2A_3} = \cos\theta|+,+,-\rangle + \sin\theta\sin\varphi|+,-,+\rangle + \sin\theta\cos\varphi|-,+,+\rangle, \tag{5}$$

or entangled GHZ-type state

$$|G(0)\rangle = \cos\vartheta|+,+,+\rangle + \sin\vartheta|-,-,-\rangle. \tag{6}$$

Separable and biserable states of atoms can be obtained from (4)–(6) by varying the parameters θ, φ, and ϑ, which determine the initial degree of entanglement of the atoms A_1, A_2, and A_3. Thus, our study covers all possible classes of entangled states.
To analyze the dynamics of entanglement, we will use two parameters: the concurrence and the fidelity. We define the concurrence in a standard way

$$C_{ij} = \max\left\{0,\ \sqrt{\lambda_1^{ij}} - \sqrt{\lambda_2^{ij}} - \sqrt{\lambda_3^{ij}} - \sqrt{\lambda_4^{ij}}\right\},$$

where $\lambda_1^{ij}, \lambda_2^{ij}, \lambda_3^{ij}$ and λ_4^{ij} are the eigenvalues of the "spin-flipped" two-atom density operator ζ_{ij} in decreasing order, and ζ_{ij} is defined as $\zeta_{ij} = \rho_{A_iA_j}(\sigma_y\otimes\sigma_y)\rho^*_{A_iA_j}(\sigma_y\otimes\sigma_y)$.
In the case of the GHZ-state, the concurrence as a criterion of atom entanglement is not very informative, since when averaging the three-atom density matrix $\hat{\rho}_{A_1A_2A_3}(t)$ over the variables of one of the atoms, the two remaining atoms turn out to be disentangled. Therefore, investigating the entanglement dynamics of the GHZ-state, we use fidelity as a measure of entanglement. The fidelity is written as follows

$$F(\hat{\rho}(0), \hat{\varrho}(t)) = Tr(\hat{\rho}(0)\hat{\varrho}(t)). \tag{7}$$

where $\hat{\rho}(0)$ is the initial three-atom density matrix, and $\hat{\varrho}(t)$ is the three-atom density matrix at subsequent time instants t. We derived the analytical formulas for elements of reduced two- and three-atom density matrices for all initial atom states of the form of Eqs. (4)–(6) and all pairs of atoms.

3. Computer modeling and results

The results of computer modeling of the pairwise concurrences $C_{ij}(\gamma t)$ for initial qubit state (4)–(5) in the case $\theta = \arccos(1/\sqrt{3})$, $\varphi = \pi/4$, and thermal field (2) are shown in Fig. 2. Figures represent the behavior of concurrences, calculated for various photon multiples m. In Fig. 3, we plot the fidelity $F(\gamma t)$ for the initial qubit GHZ-state (6) in the case $\vartheta = \pi/4$ and thermal field (2), calculated for various photon multiples m.

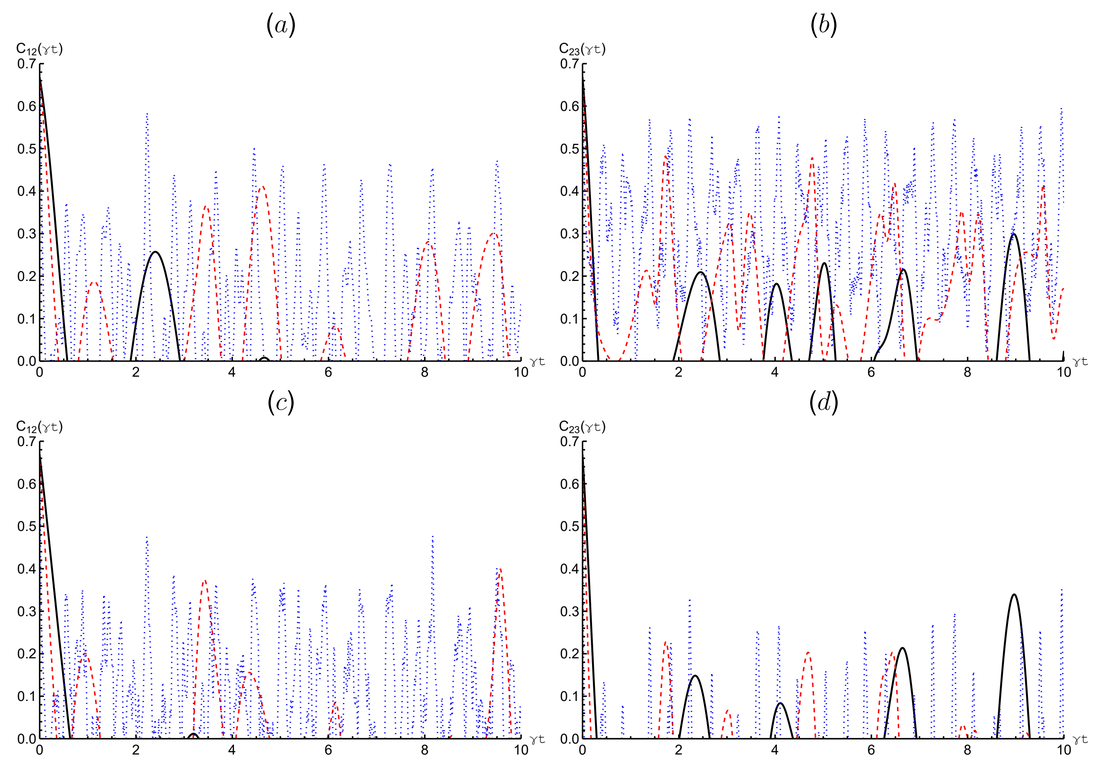

Figure 2: *The concurrence $C_{12}(\gamma t)$(or $C_{13}(\gamma t)$) (a, c) and $C_{23}(\gamma t)$ (b, d) are plotted as a function of scaled time γt for the initial W-state of the form (4) (a, b) and for the W-state of the form (5) (c, d). The mean number of photons is $\langle n\rangle = 1$. The photon multiple m: $m = 1$ (black solid line), $m = 2$ (red dashed line), $m = 4$ (blue dotted line). The initial parameters: $\theta = \arccos(1/\sqrt{3})$ and $\varphi = \pi/4$.*

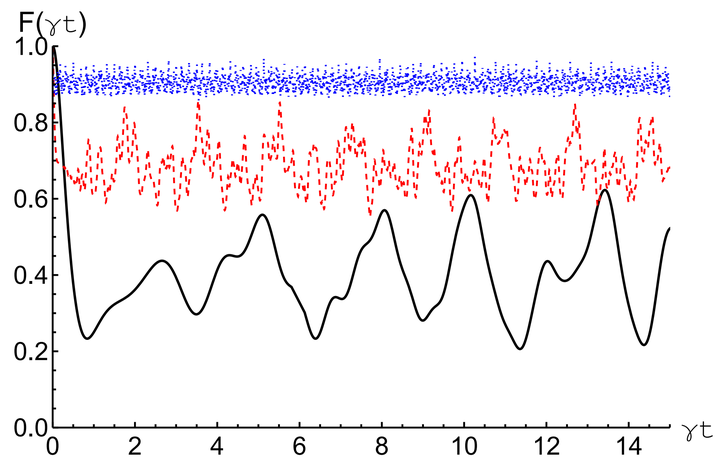

Figure 3: *The fidelity $F(\gamma t)$ are plotted as a function of the scaled time γt for the initial GHZ-state of the form (6) with $\vartheta = \pi/4$. The mean number of photons is $\langle n\rangle = 2.5$. The photon multiple m: $m = 1$ (black solid line), $m = 3$ (red dashed line), $m = 6$ (blue dotted line).*

The following conclusions can be drawn from the presented figures:

- The results of pairwise concurrence C_{ij} calculations for the initial atomic W-states (4)–(5) showed that in the nonlinear multiphoton processes the atomic entanglement is stronger than in the linear one-photon process. We also obtained that ESD takes place for small values of multiple m, and the time of ESD can be controlled by varying this parameter. For large values of m, the ESD disappears for some pairwise concurrences and remains for others at a relatively high intensity of the thermal field $\langle n\rangle$ (see Figure 2). Moreover, the W-state of the form (4) is the most resistant to the thermal noise of the resonator, for any intensities $\langle n\rangle$ of the thermal field of the resonator (2) and for any photon multiples m.
- Fig. 3 shows that for large values of photon multiples m, the oscillation of fidelity decreases, and we obtain the long-lived entangled GHZ-state (6) even for sufficiently intense fields $\langle n\rangle$ (see Figure 3). We also note that the pair entanglement of atoms in the case of the GHZ state (6) is completely absent for any intensities $\langle n\rangle$ of the thermal field of the resonator (2) and for any photon multiples m.
- An additional analysis of the 3D graphs of pairwise concurrences from the scaled time γt and initial parameters θ, φ allows us to generalize the results for W-states (4)–(5) to the case of arbitrary initial parameters θ, φ.
- Overall, this research improves our understanding of the peculiarities of entanglement dynamics in the considered three-atom JCM induced by multiphoton interaction with thermal cavity noise.