Thermal entanglement in the three-qubit Tavis-Cummings model with manyphoton transitions

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1. The model and its exact solution

We consider three identical two-level natural or artificial atoms (qubits) Q_1, Q_2, and Q_3, which are trapped in a single-mode infinite-Q resonator and resonantly interact with the mode of the thermal field via multiphoton transitions. We assume that all qubit-cavity couplings are equal, i.e., $\gamma_{Q_1} = \gamma_{Q_2} = \gamma_{Q_3} = \gamma$. The configuration of the model is shown in Fig. 1.

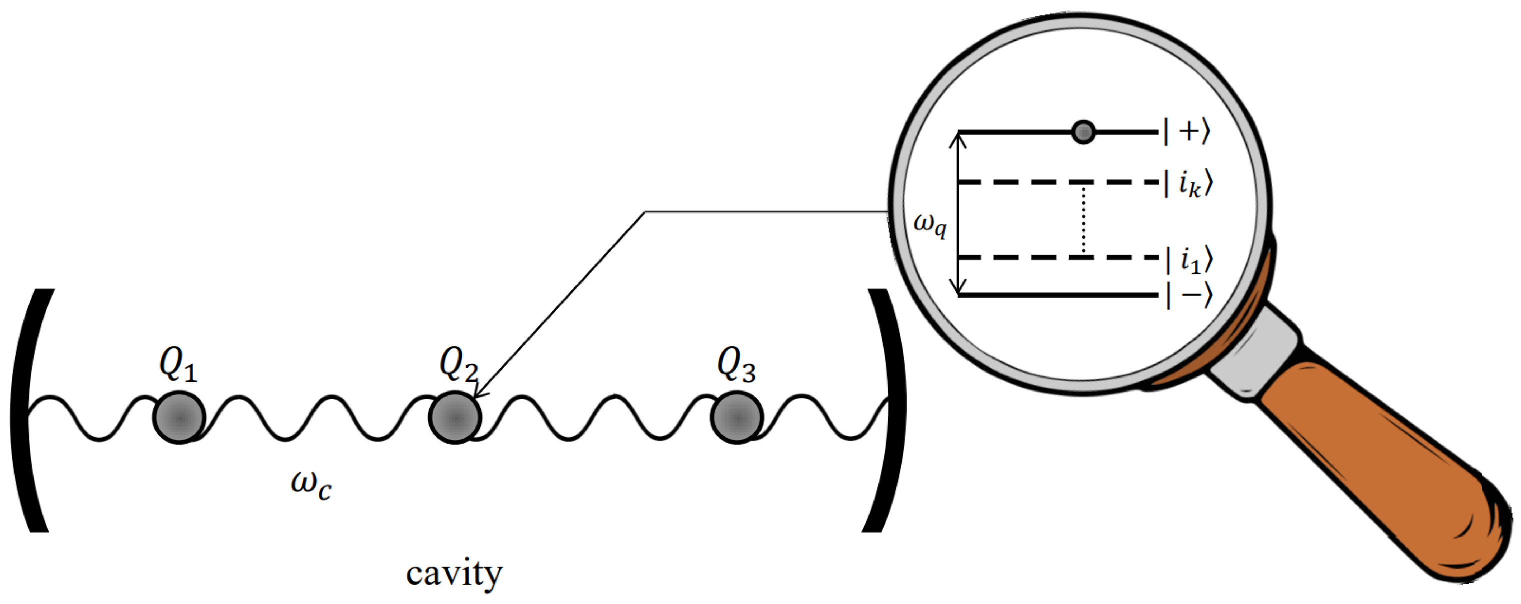

Figure 1: *Configuration of the model under study. Here, ω_c is the frequency of the resonator, $\omega_q = m\omega_c$ is the transition frequency in the qubit between the excited $|+\rangle$ and the ground $|-\rangle$ levels, m is the photon multiple of transitions, and $|i\rangle_k$ are the virtual intermediate levels, the number of which is determined by the parameter $k = m - 1$*

The Hamiltonian of the interaction of the studied model will be written in the dipole approximation and the rotating wave approximation in the following form

$$\hat{H}_I = \hbar\gamma \sum_{i=1}^{3} \left(\hat{\sigma}_i^+ \hat{a}^m + \hat{\sigma}_i^- \hat{a}^{+m} \right), \qquad (1)$$

where $\hat{a}^+(\hat{a})$ is the operator of the creation (annihilation) of photons of the resonator mode of the field, $\hat{\sigma}_i^+$ and $\hat{\sigma}_i^-$ are the raising and the lowering operators in the i-th qubit. Index i in (1) numbers the qubits trapped in a cavity.

As the initial state of the resonator field, we choose a thermal state with a density matrix of the form:

$$\hat{\rho}_F(0) = \sum_n p_n |n\rangle\langle n|. \qquad (2)$$

There are weight coefficients

$$p_n = \frac{\langle n\rangle^n}{(1+\langle n\rangle)^{n+1}}, \quad \langle n\rangle = \frac{1}{e^{\hbar\omega_c/k_B T} - 1}$$

the average number of photons of the resonator, T is the cavity temperature.

We derived the solutions of the quantum Liouville equation for the whole density matrix $\hat{\rho}_{Q_1Q_2Q_3F}$ of the considered system with Hamiltonian (1)

$$i\hbar \frac{\partial \hat{\rho}_{Q_1Q_2Q_3F}}{\partial t} = \left[\hat{H}_I, \hat{\rho}_{Q_1Q_2Q_3F} \right], \qquad (3)$$

and for initial qubits GHZ-states and the thermal state of the cavity field (2).

2. Selection of initial states and calculation of negativity, fidelity

Let's choose entangled GHZ-type states of the form as the initial states of the qubits subsystem

$$|G_1(0)\rangle_{Q_1Q_2Q_3} = \cos\vartheta|+,+,+\rangle + \sin\vartheta|-,-,-\rangle, \qquad (4)$$

$$|G_2(0)\rangle_{Q_1Q_2Q_3} = A|-,-,+\rangle + B|-,+,-\rangle + C|+,-,-\rangle + D|+,+,+\rangle, \qquad (5)$$

$$|G_3(0)\rangle_{Q_1Q_2Q_3} = A|+,+,-\rangle + B|+,-,+\rangle + C|-,+,+\rangle + D|-,-,-\rangle, \qquad (6)$$

$$|G_4(0)\rangle_{Q_1Q_2Q_3} = A|+,-,-\rangle + B|-,+,+\rangle. \qquad (7)$$

Separable, biserable, and W-states of qubits can be obtained from GHZ-states by varying the parameters θ, φ, ϑ, A, B, C, and D, which determine the initial degree of entanglement of the qubits Q_1, Q_2, and Q_3. Thus, our study covers all possible classes of entangled states.

To analyze the dynamics of entanglement, we will use two parameters: the negative criterion and the fidelity. We define the negativity in a standard way

$$\xi_{Q_iQ_j}(t) = -2\sum_l \lambda^-_{l,ij}, \qquad (8)$$

where $\lambda^-_{l,ij}$ are the negative eigenvalues of the two-qubit density matrixes partially transposed in variables of one qubit $\hat{\rho}^{T_1}_{ij}$.

In the case of the GHZ-state, the pairwise negativity as a criterion of qubit entanglement is not very informative, since when averaging the three-qubit density matrix $\hat{\rho}_{Q_1Q_2Q_3}(t)$ over the variables of one of the qubits, the two remaining qubits turn out to be disentangled. Therefore, investigating the entanglement dynamics of the GHZ-state, we use fidelity as a measure of entanglement. The fidelity is written as follows

$$F(\hat{\rho}(0), \hat{\varrho}(t)) = Tr(\hat{\rho}(0)\hat{\varrho}(t)). \qquad (9)$$

where $\hat{\rho}(0)$ is the initial three-qubit density matrix, and $\hat{\varrho}(t)$ is the three-qubit density matrix at subsequent time instants t. We derived the analytical formulas for elements of reduced two- and three-qubit density matrices for all initial qubit states of the form of Eqs. (4)–(7) and all pairs of qubits.

3. Computer modeling and results

The results of computer modeling of the pairwise negativities $\xi_{Q_iQ_j}(\gamma t)$ for initial qubit state (5)–(6) in the case $A = B = C = 1/\sqrt{3}$, $D = 0$, and thermal field (2) are shown in Fig. 2. Figures represent the behavior of negativities, calculated for various mean photon numbers $\langle n\rangle$ (a, b) and the photon multiple m (c, d). In Fig. 3, we plot the fidelity $F(\gamma t)$ for the initial qubit GHZ-states (4)–(7) in the case of maximally symmetric states and thermal field (2), calculated for various photon multiples m.

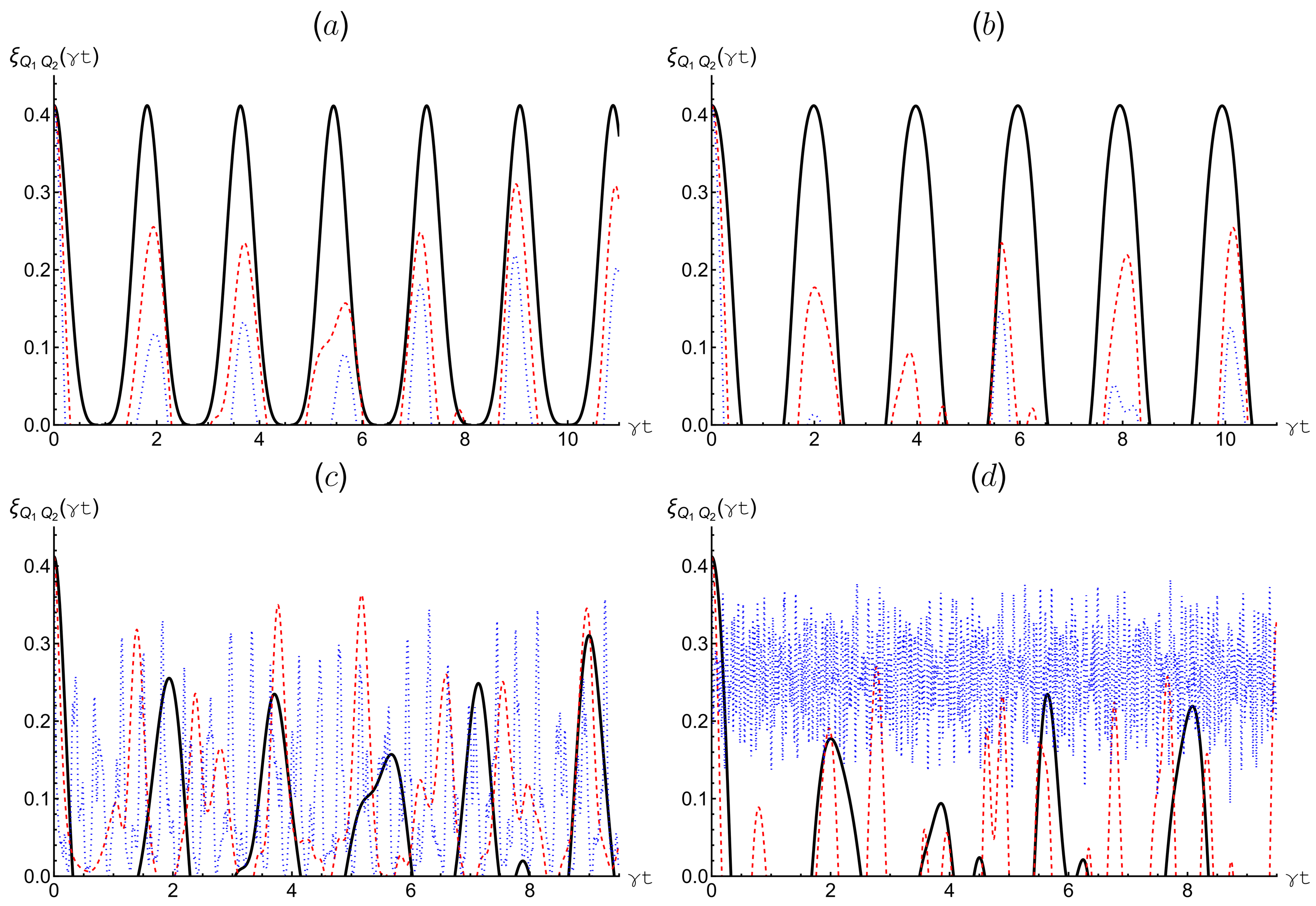

Figure 2: *The negativity $\xi_{Q_iQ_j}$ as functions of the scaled time γt for the initial state of the form (5) (a, c) and (6) (b, d) in the case $A = B = C = 1/\sqrt{3}$, $D = 0$. The photon multiple m and the mean number of photons $\langle n\rangle$ on the graphs (a, b): $m = 1$ and $\langle n\rangle = 0.001$ (black solid line), $\langle n\rangle = 1$ (red dashed line), $\langle n\rangle = 2.5$. On the graphs (c, d): $m = 1$ (black solid line), $m = 2$ (red dashed line), $m = 4$ (blue dotted line), and $\langle n\rangle = 1$.*

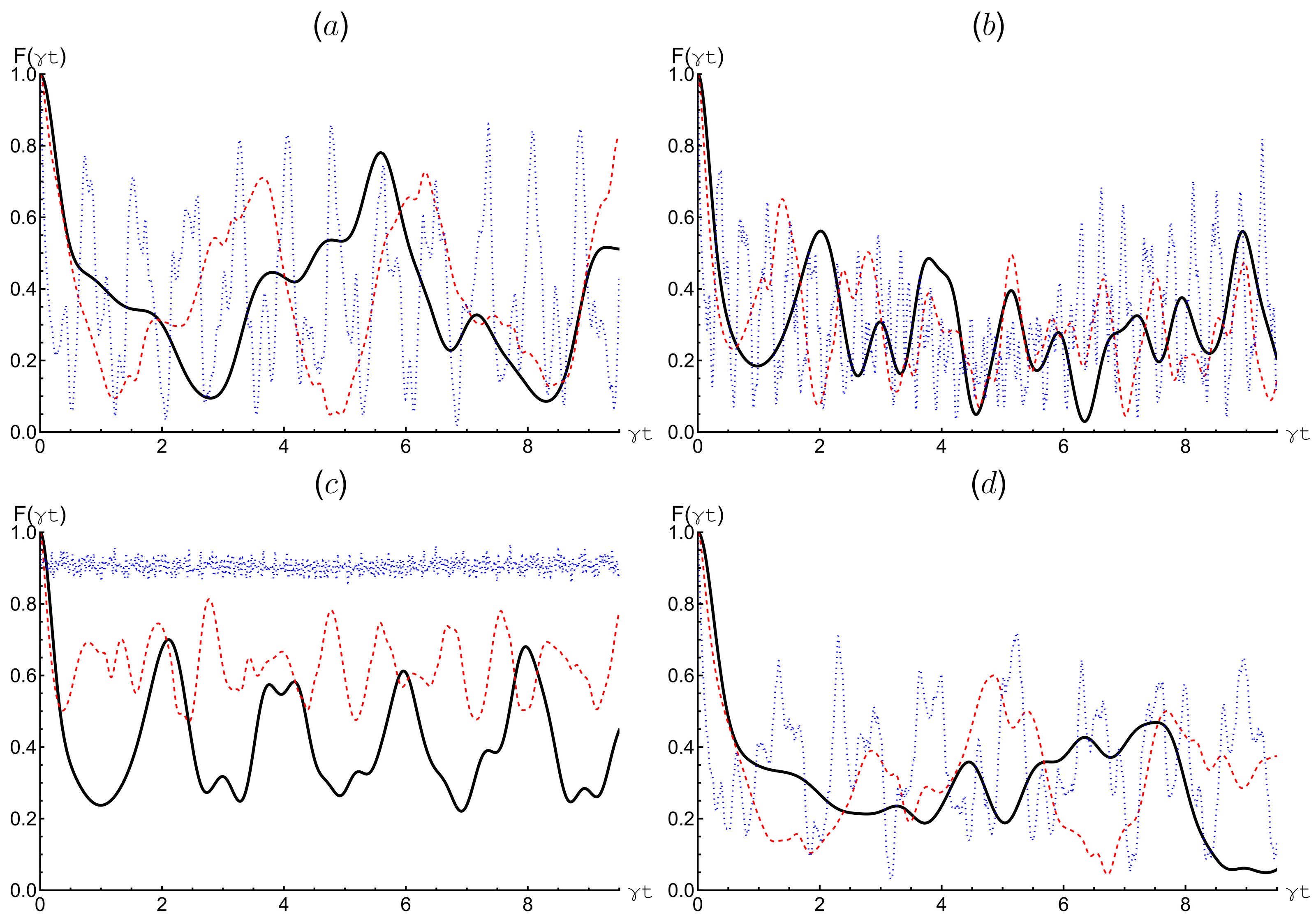

Figure 3: *The fidelity F as functions of the scaled time γt for the initial GHZ-state of the form (4) (a), (5) (b), (6) (c), and (7) (d). The mean number of photons is $\langle n\rangle = 1$. The photon multiple: $m = 1$ (black solid line), $m = 2$ (red dashed line), $m = 4$ (blue dotted line). Initial parametrs: $\vartheta = \pi/4$ (a), $A = B = C = D = 1/2$ (b, c), and $A = B = 1/\sqrt{2}$ (d).*

The following conclusions can be drawn from the presented figures:

- From figure 2, we can clearly see that the maximum degree of entanglement decreases monotonically with increasing intensity of the thermal fields of the resonators $\langle n\rangle$ for initial W-states of qubits. This conclusion is valid for initial states of any type.
- From Figures 2 and 3, it can be concluded that the photon of multiple m increases the maximum degree of entanglement (or fidelity) for any initial states (4)–(7). However, significant stabilization is only possible in the case of one W-state and one GHZ-like state of the form (6).
- An analysis of the dynamics of the negativity criterion shows that pair entanglement is completely absent in the thermal field (2) only in the case of the GHZ-state (4). The initial GHZ-like state of the form (7) has the highest pair entanglement.