Formation of light fields with variable orbital angular momentum using a diffractive spiral-toroidal lens

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Hypothesis under test

If $R > r_{max}$, the field in the focal plane has the most compact size and the highest concentration of energy, and it broadens and blurs on leaving the focal plane in either direction.
If $R < r_{max}$, the generated field keeps its structure well outside the focal region, changing only in scale.

Diffractive spiral-toroidal lens

A toroidal lens focuses a plane wave onto a ring of radius r_t instead of a point; letting r_t depend on the azimuth turns the focal curve into a broken spiral:

$$\tau(r,\varphi) = \mathrm{circ}(r/R)\exp\left[-ik\frac{(r-r_t(\varphi))^2}{2f}\right]e^{im\varphi} \qquad (1)$$

$$r_t(\varphi) = r_{max} - (r_{max} - r_0)\,\varphi/2\pi$$

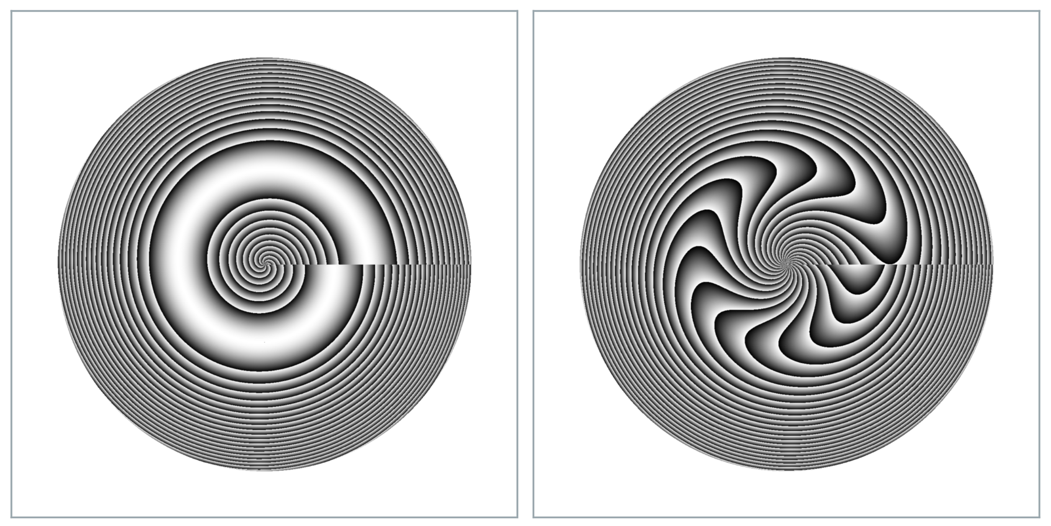
The element is thus a lens of focal length f plus an axicon whose deflection angle varies with the azimuth, and a paraxial ray leaving the radius r arrives at

$$\rho(r,z) = r\,(1 - z/f) + (z/f)\,r_t(\varphi) \qquad (2)$$

so at $z = f$ every ray lands on the design curve, while outside the focus the pattern is an affine image of it, of relative width

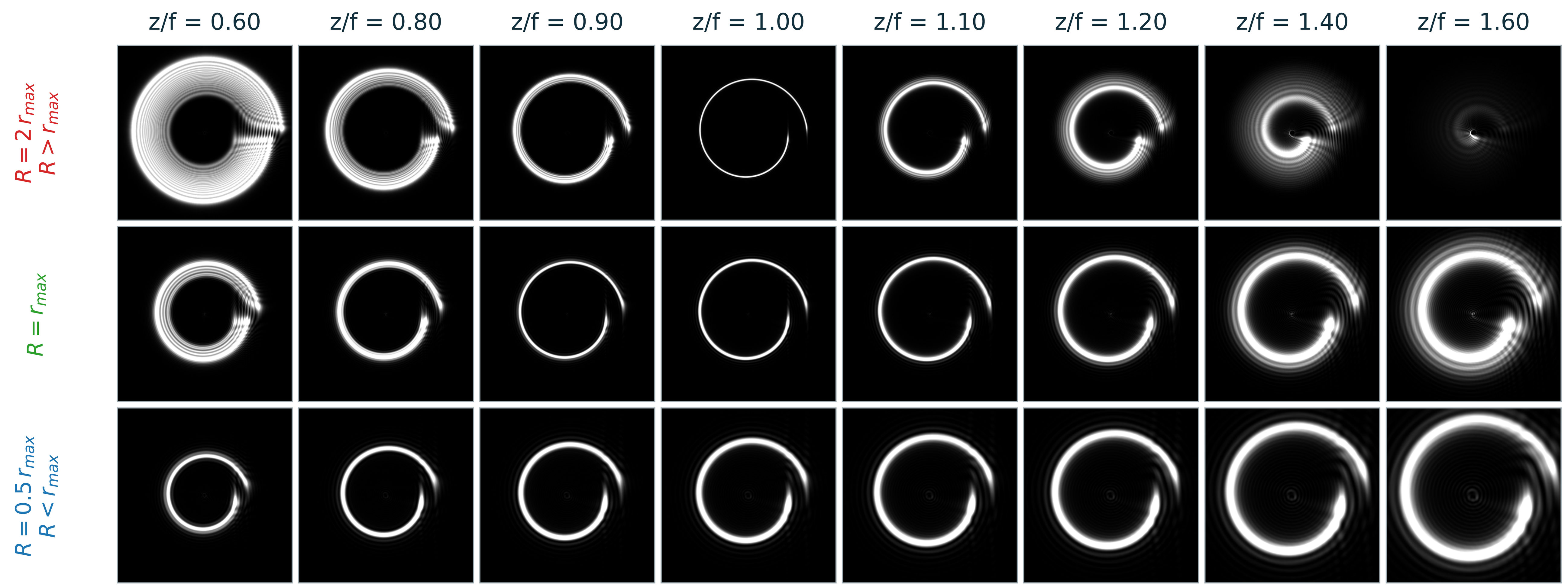
$$\Delta\rho/\rho \approx (R/r_{max})\,|f/z - 1| \qquad (3)$$

The ratio R/r_{max} alone governs the behaviour of the generated field.



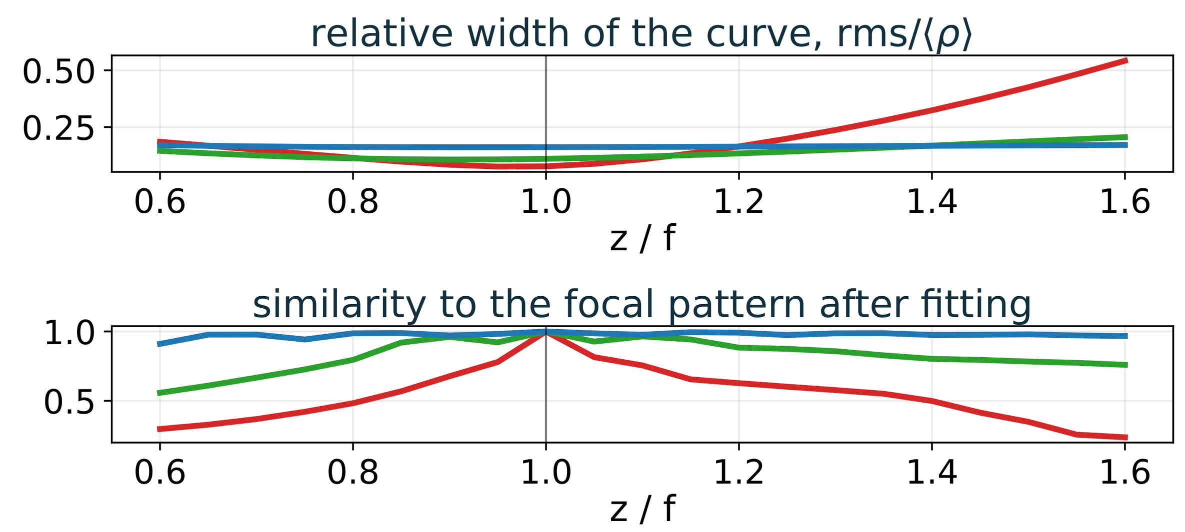
element phase, $m = 0$ — the same, $m = 15$

Longitudinal evolution



Intensity of the field formed by the spiral-toroidal lens, $m = 0$, $\lambda = 532$ nm, $f = 300$ mm, $r_{max} = 2$ mm, $r_0 = 1.34$ mm. The spiral-toroidal structure is fixed and only the aperture radius R is changed, so the design focal curve is the same in all three rows. Every panel is on one physical scale, window ±3 mm.

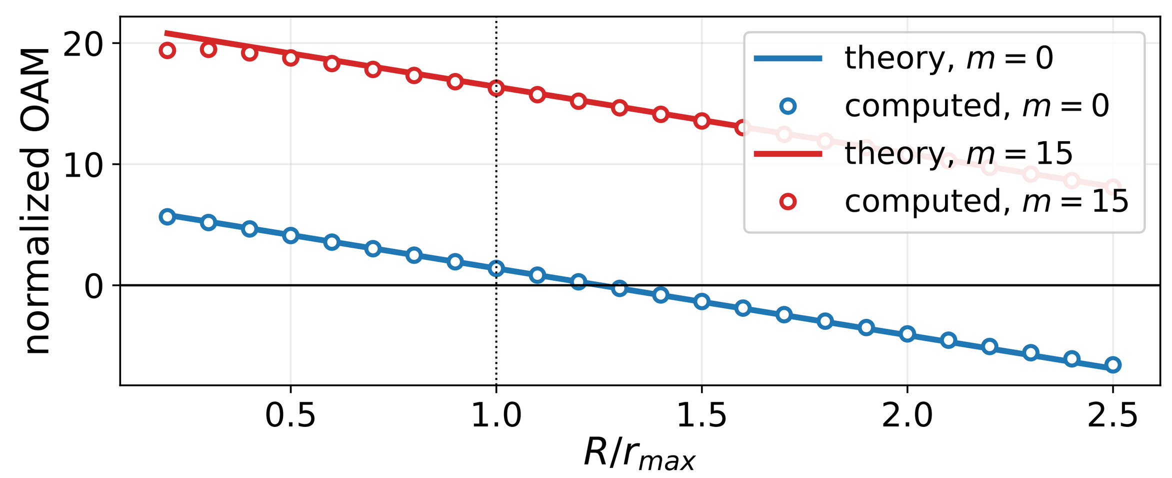
Quantitative confirmation



rms/$\langle\rho\rangle$ is the intensity-weighted rms deviation across the design curve divided by the mean radius of the pattern — a scale-free measure of blur.
$R > r_{max}$ — the most compact spot and the highest concentration of energy, $A_{eff} = 0.74$ mm^2 at $z = f$, growing 21-fold away from it; the curve survives only within ±5 % of f.
$R = r_{max}$ — the same, but over ±20 %.
$R < r_{max}$ — no pronounced focus at all: the relative width stays within 0.161...0.171 over $z/f = 0.6...1.6$ and the similarity to the focal pattern never drops below 0.91 — the field only changes scale, the fitted factor following $a \approx z/f$.
Both parts of the hypothesis are confirmed and follow from (3).

Orbital angular momentum

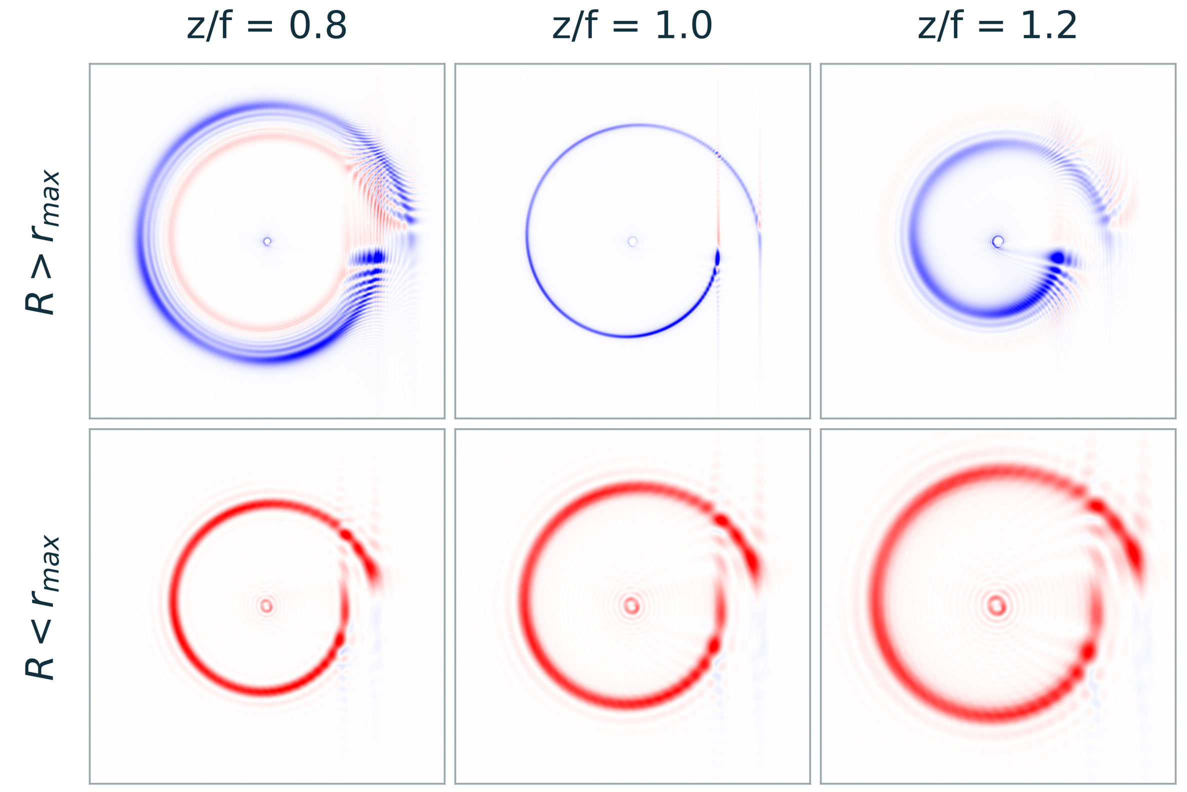
$$L = m + \frac{k\,(r_{max} - r_0)}{2\pi f}\left[\frac{r_{max} + r_0}{2} - \frac{2R}{3}\right] \qquad (4)$$



For uniform illumination ($\langle r\rangle = 2R/3$) the azimuthal phase gradient of the element gives a closed form for the normalized OAM, matching the computed values to 1...3 % over $R/r_{max} = 0.2...2.5$ and constant along z.
The structure carries OAM by itself: value and sign are set by the aperture ratio, the OAM vanishing at $R = 3(r_{max} + r_0)/4 \approx 1.25\,r_{max}$. Variable OAM is thus tuned by the aperture as well as by the vortex charge m.

Transverse energy flux

$$\vec{S}_\perp = \mathrm{Im}(u^*\nabla u) = |u|^2\nabla\varphi \qquad (5)$$

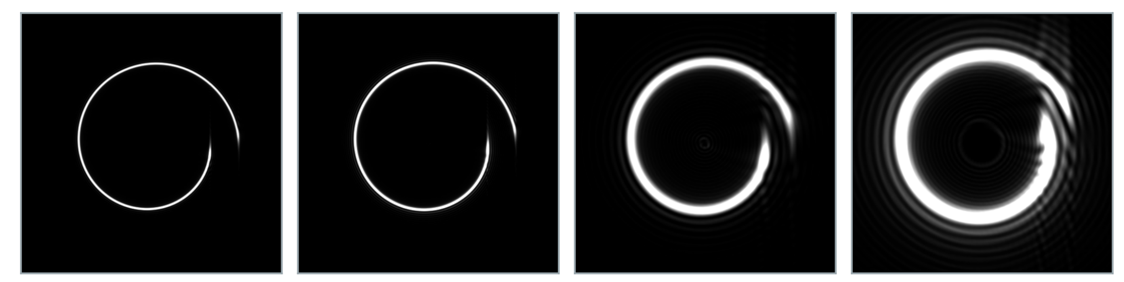


Azimuthal component S_φ: red — counter-clockwise, blue — clockwise.
At $R > r_{max}$ (top) the rotation in the focal plane is uniform, 98 % of the energy turning one way, while defocusing splits the curve into counter-rotating zones and reverses S_φ for about 20 % of it. At $R < r_{max}$ (bottom) the rotation stays one-signed and merely scales with the ring. The total OAM is conserved along z: only its density is redistributed.

Numerical model

The Fresnel integral is evaluated through a chirp-z transform, so the sampling of the observation plane does not depend on z and all planes are compared in absolute physical units.
The propagator reproduces a Gaussian beam and the first Airy zero of a circular lens to better than 1.5 %, conserves the energy to 10^{-4} and keeps the total OAM constant along z.

Effect of the vortex



$R > r_{max}$, $m = 0$ — $R > r_{max}$, $m = 15$ — $R < r_{max}$, $m = 0$ — $R < r_{max}$, $m = 15$

An added vortex of order $m = 15$ leaves the focal curve where it is and only broadens it slightly. It shifts the OAM by m, while the role of R/r_{max} is unchanged.

Applications

- optical trapping and manipulation of particles
- laser structuring of surfaces of photosensitive materials
- quantum communications

References

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