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Notes in the margins of the proof of the Fermat great theorem

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Abstract
The article examines the geometric approach to the proof of the Fermat Great Theorem (TF). Its geometric formulation is given, which consists in the fact that the trajectory on the Cartesian plane corresponding to the TF does not pass through any rational point. The equation of motion of a material point along the trajectory is obtained. The non-computability of the problem of determining the position of a point on the trajectory is shown.

Keywords: Fermat great theorem, metric, non-computability, complex dynamics

1. Introduction
Fermat Great Theorem attracted the attention of mathematicians all over the world for more than 300 years from the moment of its formulation by P. Fermat in 1637 until its proof by Andrew Wiles in 1994-1995 [1, 2]. Both before this outstanding event and after it, attempts to find the simplest possible proof of this theorem did not stop, as if stimulated by Fermat's remark left in the margins of Diophantus' Arithmetic that he "managed to find a truly amazing proof", which he cannot cite for the lack of space [3].
The searches for proof of the Fermat theorem led to important results in the modern number theory. This has a deep meaning that indicates the relationship of seemingly distant fields of knowledge. It is this that gives meaning to the search for alternative proofs of the already proven theorem, allowing us to reveal new relationships that previously escaped the attention of researchers.
This article is devoted to this aspect of the issue. To study Fermat theorem, we use methods and concepts that are far from algebraic, which were used for the investigation of the ways to prove the theorem earlier.

2. Geometric formulation of the Fermat theorem
As shown in [3] Fermat theorem (TF) can be formulated geometrically in the form of the following statement:

"A curve on the plane given by the equation

$$x^n + y^n = 1, n \ge 3 \tag{1}$$

does not contain a single rational point"

Here *x,y* – Cartesian coordinates of plane points, *n* – is an integer. In polar co-ordinates $x = r \cdot \cos\varphi$, $y = r \cdot \sin\varphi$ the equation of the curve looks as follows

$$r = R(\varphi) = \left(\cos^n \varphi + \sin^n \varphi\right)^{-1/n} \tag{2}$$

A rational point is a point whose Cartesian coordinates are expressed in rational numbers. Let's try to prove this statement.
Let's imagine that a material point moves along the curve (2). We want to get the equations of its motion. To do this, consider the Euclidean metric on the Cartesian plane [4].

$$ds^2 = c^2dt^2 - dr^2 - r^2d\varphi^2 \tag{3}$$

s – is an interval on the plane, t – is a coordinated time, c – is a speed of light in the vacuum.
Substituting (2) into (3), we get the induced metric on the curve (2)

$$(ds')^2 = c^2dt^2 - \left[R^2(\varphi) + \left(\frac{dR}{d\varphi}\right)^2\right]d\varphi^2 \tag{4}$$

s'- is an interval on the curve. Since all further arguments and calculations will concern the curve (2), we will omit the «′» sign. In accordance with (4) the metric tensor g_{ik} on the curve (2) looks as follows (i,k = 0, 1)

$$g_{ik} = \begin{pmatrix} 1 & 0 \\ 0 & -\left[R^2(\varphi) + \left(\frac{dR}{d\varphi}\right)^2\right] \end{pmatrix} \tag{5}$$

The equations of motion of a point along the curve (2) have the form

$$\begin{aligned} &\frac{d^2x^i}{ds^2} + \Gamma^i_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds} = 0 \\ &\Gamma^i_{kl} = \frac{1}{2}g^{im}\left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m}\right) \end{aligned} \tag{6}$$

Γ^i_{kl} – Christoffel symbols [4], $x^0 = ct$, $x^1 = \varphi$. Using (5) we receive

$$\begin{aligned} &\frac{d^2t}{ds^2} = 0 \\ &\frac{d^2\varphi}{ds^2} + \frac{1}{2}\frac{\partial}{\partial\varphi}\ln|g_{11}|\left(\frac{d\varphi}{ds}\right)^2 = 0 \end{aligned} \tag{7}$$

It follows from the first equation (7) that the coordinate time t linearly depends on the proper time s and with the help of a choice of units it can be put as $t = s$. The second equation by a single integration can be reduced to the form

$$\frac{d\varphi}{ds} = \frac{\Omega}{\sqrt{|g_{11}(\varphi)|}}, \Omega^2 = \left(\frac{d\varphi}{ds}\right)^2\left[R^2(\varphi) + \left(\frac{dR}{d\varphi}\right)^2\right] \tag{8}$$

Ω – is the first integral of the second equation (7). Equation (8) can easily be solved numerically. Figures 1 and 2 below show the results.

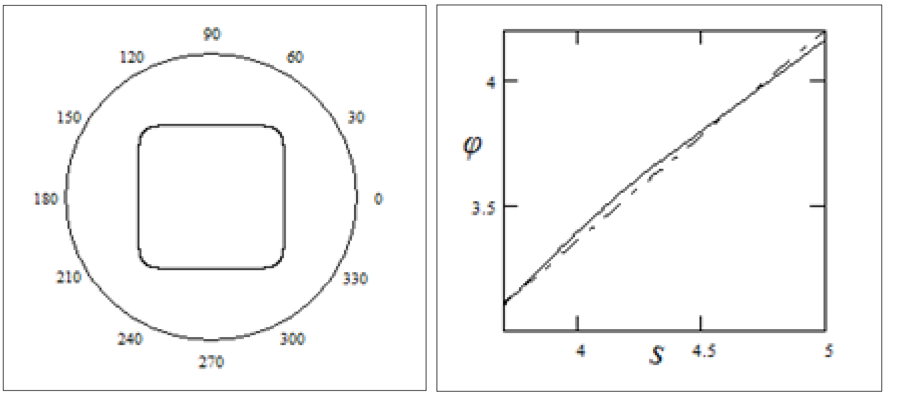

Fig 1: Left: the trajectory (2) $r = R(\varphi)$ for $n = 8$; Right - the solution of the equation (8) $\varphi(s)$ (line) and the trend line (dot-dashed); $\Omega = 1$.

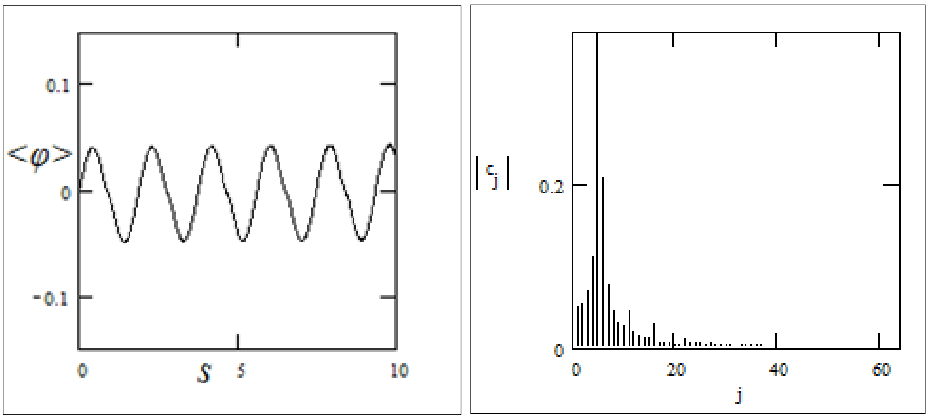

Fig 2: Left: <$\varphi(s)$> - solution of equation (8) minus the trend; Right: its Fourier spectrum c_j, calculated by the discrete fast Fourier transform method, $0<j<256$; $\Omega = 1$.

3. Discussion

The most interesting information is contained in the spectrum of the solution <φ (s)> minus the linear trend, shown in Figure 2 on the right. It contains a component corresponding to the fundamental frequency ω_0 of the solution (it has a maximum amplitude), the regions $\omega > \omega_0$ to the right of it, where harmonics $\omega_m = m\omega_0$ (m is an integer) are distinguished, multiples of the fundamental and combinational harmonics having a modulation nature. This corresponds to the nonlinear character of equation (8).

The region $\omega < \omega_0$ to the left of the fundamental frequency corresponds to a motion with frequencies less than the fundamental, and its explanation requires additional considerations. Note that the trajectories for all even $n \geq 4$, have a finite symmetry group including a subgroup C_4 [5], whose elements are rotations to the angles $\pi/2$, π, $3\pi/2$, 2π. This can lead to a change in the movement period. It can be assumed that these movements are associated with rotations of the trajectory as a whole by an angle multiple of $\pi/2$, which are not noticeable to a remote observer. For example, if a particle makes a complete revolution in time $T = 2\pi/\omega_0$, and if the trajectory turns in the direction of the particle movement by half a revolution, then the remote observer will perceive this as the movement of a particle with a doubled period $2T$. The turns of the trajectory against the direction of the particle's movement from the point of view of a remote observer look like rotations of a particle with a higher frequency.

There are two circumstances in favor of this assumption. First, the described phenomenon is absent for $n = 2$, when the trajectory is a circle and the symmetry group of the trajectory becomes continuous. Secondly, this phenomenon is absent when the equation $r = R(\varphi)$ does not have a symmetry group at all (for example, for $r = k\varphi$, k – is a real number) [1].

By itself, the presence of a continuous frequency spectrum indicates the complex dynamics of the system, leading to its unpredictable behavior over long periods of time. For us, it is important the non-computability of the results of such a movement [6].

Let's imagine that we have at our disposal a supercomputer capable of overcoming the Turing barrier [7], which we use to calculate the position of a point on the trajectory of motion. Due to the non-computability of the trajectory on an ordinary Turing machine, the result of calculating the coordinates of the trajectory on a supercomputer for any point of the trajectory will be expressed in numbers that are not rational, which proves the original statement.

Since the symmetry groups for trajectories for other values of even n have the same finite subgroup C_4, the result will be true for them as well. For odd n, the symmetry group will be different [3], but the reasoning will not change qualitatively.

Finally, we note that the value of $n = 8$, as well as the form of representation of the spectrum graph in Fig. 2, are chosen only for reasons of clarity and are not principal.

4. Conclusion

The article examines the geometric approach to the proof of Fermat's Great Theorem (FT). A geometric formulation of FT is given, which consists in the fact that the trajectory on the Cartesian plane corresponding to the FT does not pass through any rational point. This trajectory is invariant under some symmetry operations which conserve its form. The equation of motion of a material point along the trajectory is obtained and solved numerically. The Fourier spectrum of this solution demonstrates some features which are inherent to the complex dynamics of motion and means unpredictability i.e. non-computability of the position of a point in the trajectory. This in turn means that coordinates of all the points of the trajectory don't have rational values, which proves the TF.

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