Coherence-enhanced entanglement between isolated atom and Jaynes-Cummings atom

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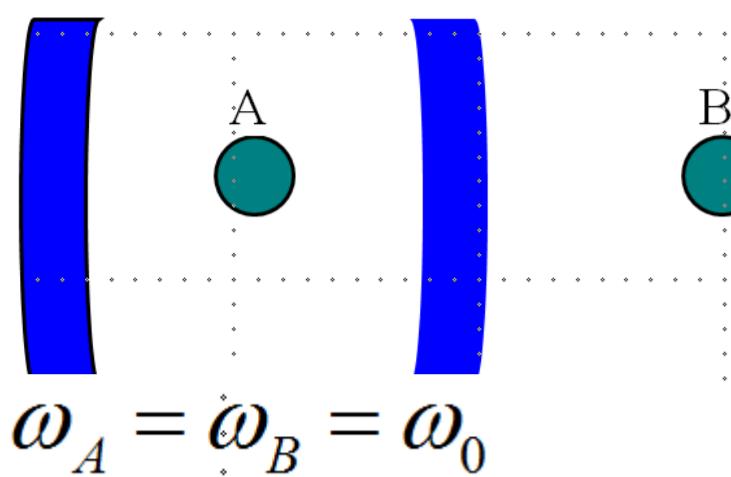
Model and negativity calculations

Hamiltonian in the interaction picture

$\omega_A = \omega_B = \omega_0$

$$\hat{H} = \sum_{i=1,2} \hbar\Delta\hat{\sigma}_z^i + \hbar g(\hat{\sigma}_A^+\hat{a} + \hat{\sigma}_A^-\hat{a}^\dagger) + \hbar X \left(\hat{a}^\dagger\right)^2 \hat{a}^2 + \hbar J(\hat{\sigma}_A^+\hat{\sigma}_B^- + \hat{\sigma}_B^+\hat{\sigma}_A^-),$$

Initial separable incoherent atomic states

$$|\Psi(0)\rangle_A = \cos\theta_1 |+\rangle + \sin\theta_1 |-\rangle \quad (1) \quad \text{or} \quad |\Psi(0)\rangle_B = |\Psi(0)\rangle_A = \cos\theta_2 |+\rangle + \sin\theta_2 |-\rangle \quad (2)$$

Initial thermal cavity field state

$$\hat{\rho}_F(0) = \sum_n p_n |n\rangle\langle n|, \quad p_n = \frac{\bar{n}^n}{(1+\bar{n})^{n+1}}, \quad \bar{n} = \left(\exp[\hbar\omega/k_B T]-1\right)^{-1},$$

Time-dependent density matrix

$$\hat{\rho}(t) = \sum_{i=1}^{4}\sum_{n=0}^{\infty} p_n |\Psi_{in}(t)\rangle\langle\Psi_{in}(t)|,$$

$$|\Psi_{in}\rangle = C_{i1n}|-,-,n+2\rangle + C_{i2n}|+,-,n+1\rangle + C_{i3n}|-,+,n+1\rangle + C_{i4n}|+,+,n\rangle$$

Reduced atomic density matrix for initial atomic states (1) or (2)

$$\hat{\rho}_{AB}(t) = [Tr_F\rho(t)] = \begin{pmatrix} U(t) & 0 & 0 & 0 \\ 0 & W(t) & H(t) & 0 \\ 0 & H^*(t) & V(t) & 0 \\ 0 & 0 & 0 & R(t) \end{pmatrix}$$

Matrix elements for initial atomic state (1)

$$U = \sum_{n=0}^{\infty} p_n |C_{41,n}(t)|^2, \quad V = \sum_{n=0}^{\infty} p_n |C_{21,n}(t)|^2, \quad W = \sum_{n=1}^{\infty} p_n |C_{31,n-1}(,t)|^2, \quad R = \sum_{n=1}^{\infty} p_n |C_{11,n-1}(,t)|^2,$$

$$\rho_{23} = \sum_{n=1}^{\infty} p_n C_{21,n-1}(t) C_{31,n-1}(t)^*.$$

The negativity

$$\varepsilon = -2\sum_i \mu_i^-.$$

Here μ_i^- is the eigenvalues of the partial transpose matrix

Simulations

(a) (b)
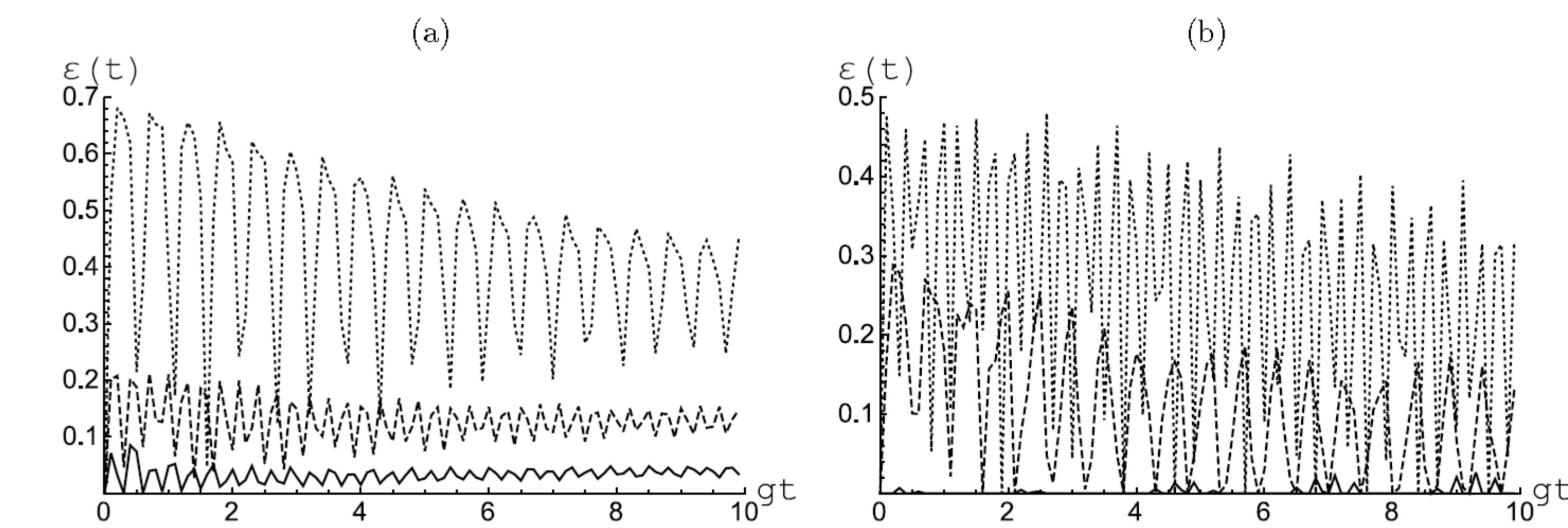
Figure 1. The negativity as a function of a scaled time gt for initial atomic state $|+,-\rangle$ (a) (the detuning $\delta = 5$ (solid), $\delta = 10$ (dashed) and $\delta = 50$ (dotted)) and coherent initial atomic state (2) with $\theta_1 = \pi/4$, $\theta_2 = -\pi/4$ (b) (the detuning $\delta = 10$ (solid), $\delta = 50$ (dashed) and $\delta = 100$ (dotted)). The strength of dipole interaction $\alpha = 0.1$. The mean photon number $\bar{n} = 20$.


(a) (b)
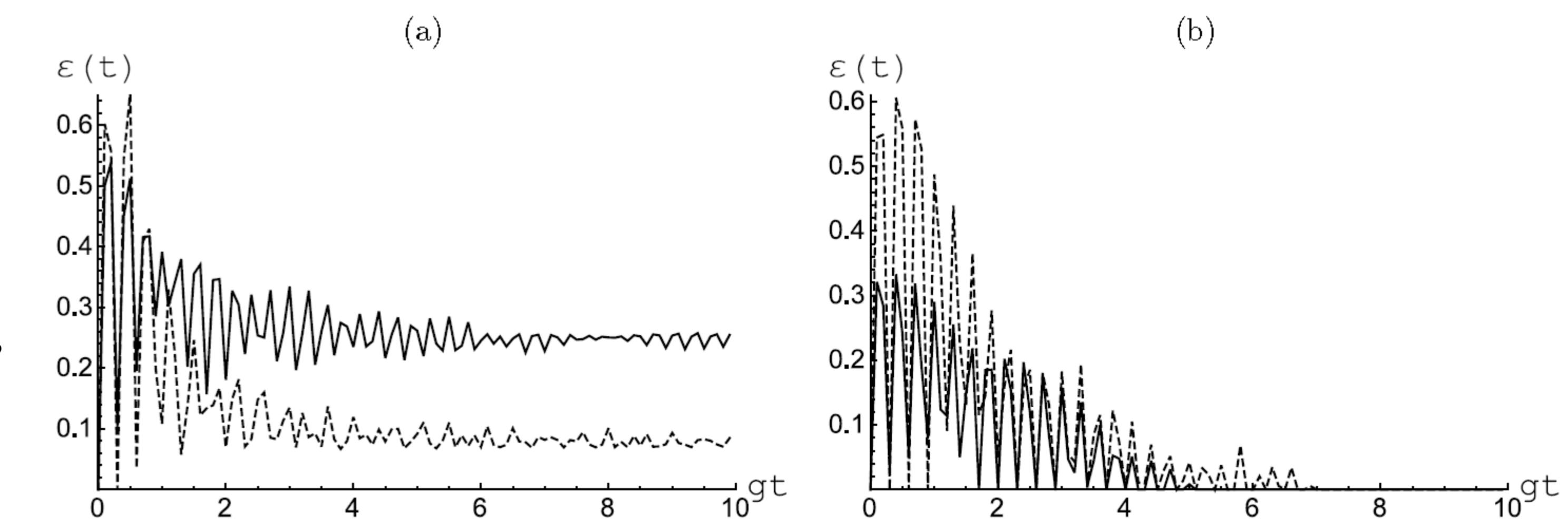
Figure 3. The negativity as a function of a scaled time gt for initial atomic states: $|+,-\rangle$ (a) (the dipole-dipole strength $\chi = 0.1$ and the detuning $\delta = 5$ (solid), $\delta = 10$ (dashed) and $\delta = 50$ (dotted)) and coherent initial atomic state (2) with $\theta_1 = \pi/4$, $\theta_2 = -\pi/4$ (b) (the detuning $\delta = 10$ and the strength of dipole interaction $\alpha = 0.5$ (solid) and $\alpha = 5$ (dashed)). The mean photon number $\bar{n} = 10$.

$$\delta = \Delta/g, \chi = X/g, \alpha = J/g,$$